Thinking about Mathematics: The Philosophy of Mathematics

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This unique book by Stewart Shapiro looks at a range of philosophical issues and positions concerning mathematics in four comprehensive sections. Part I describes questions and issues about mathematics that have motivated philosophers since the beginning of intellectual history. Part II is an historical survey, discussing the role of mathematics in the thought of such philosophers as Plato, Aristotle, Kant, and Mill. Part III covers the three major positions held throughout the twentieth century: the idea that mathematics is logic (logicism), the view that the essence of mathematics is the rule-governed manipulation of characters (formalism), and a revisionist philosophy that focuses on the mental activity of mathematics (intuitionism). Finally, Part IV brings the reader up-to-date with a look at contemporary developments within the discipline. This sweeping introductory guide to the philosophy of mathematics makes these fascinating concepts accessible to those with little background in either mathematics or philosophy.

Thinking about Mathematics: The Philosophy of Mathematics Details

Date	:	Published October 5th 2000 by Oxford University Press, USA (first published January 1st 2000)
ISBN	:	9780192893062
Author	:	Stewart Shapiro
Format	:	Paperback 328 pages
Genre	:	Science, Mathematics, Philosophy, Nonfiction, Logic

From Reader Review Thinking about Mathematics: The Philosophy of Mathematics for online ebook

Joost says

An introduction to the philosophy of mathematics that touches upon most relevant philosophies, old and new - with perhaps only a significant gap in truly contemporary philosophies of mathematical practice. It lacks some depth, but that is to be expected of an introductory book. All in all, it seems like a good book to teach a philosophy of math course from :)

Anju Mai says

I read this once in a study group. It was pretty difficult but many parts were interesting. I would have preferred a more popularized book about the topic. It was very scientific. I discovered that I am a Platonist concerning the nature of mathematics, that is, I believe mathematics to be something that truly exists, has always existed and will always exist in its own realm. There exist such things as perfect circles, for example, in the world of ideas. Also, the universe is governed by mathematics (instead of, perhaps, man's free will - or an animal's). But is that even debatable?

Tyler Minix says

This book is a must read for anyone whom concerns themselves with the philosophy of mathematics; especially if you're a student of a highly mathematical field-i.e. Engineering, Physics, Mathematics- the value this book could possess for you will be incalculable -(yes, pun intended). The concept material can be difficult, but I can grasp it with my minimal background understanding of complex mathematics and philosophy. If you -(whomever "you" might be) decide to give this book a gander, you'll not be disappointed. :)

Arax Miltiadous says

το βιβλ?ο ε?ναι για χρ?νια μελ?τη. και αξ?ζει τον κ?πο του στο ?πακρο

WarpDrive says

This is a very informative and interesting introductory book about the most critical themes of philosophy of mathematics.
Mathematics is central to epistemological analysis: it plays a central role in virtually every scientific effort aimed at understanding the material world. One of the most important questions of philosophy of mathematics is: what is it about the universe that allows mathematics such a central role in understanding it ? The answer to such foundational question requires familiarization with fundamental concepts and perspectives, such as “ontological realism” (do mathematical structures exist as separate independent entities

?) and “truth-value realism” (is there an objective truth behind logical/mathematical statements ?). And the author explains such concepts and philosophical perspectives in a very clear and lucid manner.
In any case, regardless of individual philosophical positions, the author correctly states that "it is incumbent on any complete philosophy of mathematics to account for the at-least apparent necessity and a-priority of mathematics. It must also account of the deep and intimate relationship between mathematics and science.”
In fact, the author starts his analysis by exploring the close relationship between science and mathematics, and how the scientific language is thoroughly intertwined with the mathematical language. From this, it appears clear that if we assume some sort of realism in truth-value for science (science statements can say something truthful in relation to an objective reality) then we are led to realism in truth-value for mathematics.
The author highlights that, clearly, there are deep questions to be answered:
- why is that areas of pure mathematics find unexpected applications in science long after their mathematical maturity ?
- as S.Weinberg put it: "it is positively spooky how the physicist finds the mathematician has been there before him or her"
- and Richard Feynman stated: "I find it quite amazing that it is possible to predict what will happen by mathematics, which is simply following rules which really have nothing to do with the original thing".
Simply summarized, the overall question is: why is that empirical reality fits itself into mathematical forms ?

The author makes a very compelling example that would appear to support the “ontological realist” (also called “Platonist”) answer: "impredicative" statements (definitions/statements which refer not directly to a specific entity, but to a collection that contains the defined entity). This type of definition is widely used in mathematics: example of such definition is the "least upper bound".
Godel used this very example to support his own mathematical ontological realism.
Godel showed that the only way to avoid a circular logical loop (one cannot CONSTRUCT a non-circular definition of impredicative statements), is to posit a priory independent existence of the corresponding set of mathematical objects.

After this initial overall discussion of the main themes, the author now gets into a historical overview of the different positions held by important philosophers. He starts by dealing with the ancient Greek philosophy: Plato and Aristotle. I am not overly impressed by this chapter: the treatment of these two philosophers is quite superficial. Plato's thought is almost caricatural, and Aristotle's view is very partially explained (in particular, his fascinating treatment of the concept of continuity, and of potential versus actual infinity, are ignored).

But later on the author redeems himself: there is a very nice presentation of Kant's famous position: mathematics is knowable a priori (independent of experience) and synthetic (it adds new knowledge, it is not tautological).
According to Kant, we structure our perception according to structures supplied by the human mind. It is the mind's structuring of perception that makes empirical experience possible. And mathematics is part of this framework of perception.
Part of the framework of perception that our mind provides (called by Kant "pure intuition") is the spatial-temporal form in which we position our experience: our mind processes raw perceptual information in such a way as to locate the perceived object-event in a particular space, time, and even causal relationship to other object/events. Our mind plays an active role in the framing of perceptual experience. And, according to Kant, our mathematical/geometrical intuition is part of our mind's structuring of experience.
Kant's position is very interesting and it has proved influential. But, as correctly highlighted by the author, the problem with Kant's view rises with the new non-Euclidean geometries: Kant ties our mathematical "intuition" to sense perception, so when this link is broken, Kant's approach becomes problematic.
And even if we assume that Kant's position is not linked to Euclidean geometry, it “would be curious that, in a system like Kant's, which has the ambition of delimiting once and for all the a priori presuppositions of experience, what is knowable a priori changes in response to scientific developments. And clearly there are

important concepts (such as the distinction between continuity and differentiability) that clearly have no basis in intuition. And other branches of pure and applied mathematics go even further in severing the tie with intuition."

After dealing with Kant, the author then successfully carries out a devastating critique of the empiricist approach, in particular of Mill's position that all mathematical knowledge is based on inductive generalizations from experience, whereby geometric objects are approximations of actual drawn figures. Mill held that arithmetic appears to be necessary because the axioms and definitions are "known to us by early and constant experience”.
The author also highlights that Mill's account is severely limited in scope, and it can't account for even basic principles such as mathematical induction, and concepts of infinity (for example, how can we, simply from experience-based induction, deal with the principle that every bounded set of real numbers has a least upper bound?).
Finally, the author highlights that naive induction from experience is simply untenable, especially in the light of modern physics, which indicate that the Universe operates in way that we find inconceivable. In the end we simply can't rely on the "early and constant experience" expressed by Mill.

The author then deals with the ill-fated "logicist" approach. This view posits that all mathematics can be ultimately reduced to logic. The most famous exponent of this position is B. Russell.
This view fell apart with the famous Godel's theorem, demonstrating that there are, in every sufficiently rich formal system, undeniably true statements that nevertheless cannot be proved from within the system itself.
The next philosophical position investigated in the book is "formalism", whose view is that the essence of mathematics is pure manipulation of characters, just a manipulation of linguistic symbols according to certain rules. Frege easily destroyed this formalist view, highlighting the many problems with this position: the fact that it only captures mathematical calculations and not propositions, the fact that it can't easily deal with non-finitary objects (such as pi), the fact that it can't explain why arbitrarily selected axioms and rules have proven so remarkably useful in understanding nature. According to Frege and Godel, mathematical language has MEANING and it is a gross distortion to attempt to ignore this meaning. At best, formalism focus on just one small, limited aspect of mathematics, deliberating leaving aside what is essential to the enterprise.

The next position is the so-called "intuitionism": mathematics is only but a mental "construction". This is, in my opinion, so implausible as to verging on the plain ridiculous. According to this revisionist view, fundamental aspects of logical and mathematical reasoning are inadmissible: concepts such as the principle of the "excluded middle", impredicative statements, and complete infinite collections of objects such as "the set whose elements are the continuous functions of a variable", and even discontinuous functions are meaningless, according to this philosophical perspective. Never mind that this concepts have been successfully used in modern science, up to the point of being indispensable.

The author then summarizes the main modern and contemporary perspectives:
- on one side we have the Platonist interpretation, as supported to various level by Godel, Putnam and Quine
1) Godel argues that potential circularity of self-definitions (including the famous Russell's paradox) as well as impredicative statements, which are potentially troublesome in a constructivistic perspective of mathematics, would present no problems if we posit that mathematical objects are independent of our constructions. For a realist on ontology, a definition of a mathematical object is not a recipe for creating such object, but simply a method for describing or pointing to an already existing entity. So, from that perspective, impredicative definitions are innocuous and present no conceptual problem. Godel also posits that we have some “perception” of mathematical objects, as is seen for example from the fact that axioms “force on us as being true” (this appears to be quite a Kantian perspective). Godel also makes an interesting analogy between physics and mathematics: we learn about physical objects via highly theoretical scientific activity, and although scientific theories must be connected to observation, they go well beyond observation, by creating models. Do we “see” the wave function in quantum mechanics ?

2) Quine accepts mathematics as true for the same reason that physics is true. Indeed, for Quine mathematics has the same status as the more theoretical parts of science. Quine also highlights that it is hard to draw a sharp, principled boundary between mathematics and the more theoretical branches of science.
3) Putnam highlights that classical and modern physics are full of magnitudes that are measured with real numbers: volume, force, pressure etc. Moreover, the relations between these magnitudes are expressed in equations. Thus, there is simply no hope of “doing” science without mathematics, and this clearly strongly points to the ontological reality of mathematical objects. The author also highlights correctly that explanations of physical phenomena sometimes directly require and involve purely mathematical facts: why does rain form into drops ? Because of the surface tension, and because of the purely mathematical fact that a sphere is the largest volume that can be enclosed with a given surface. Such examples are recurrent in particles physics and quantum mechanics (actually the more fundamental the level, the more intimately connected are purely mathematical properties such as symmetry and the physical reality).

- at the opposite side of the spectrum, we have ontological anti-realists, who deny the independent existence of mathematical objects:
- "fictionalism" is the idea that mathematical objects are fictional, an invention of the human mind. Field, who is probably the most famous exponent of this perspective, tried (in his book “Science without numbers”) to demonstrate that modern science does not depend on independent mathematical objects, arguing that we could use 4-dimensional space-time points as a basis for the formulation of scientific laws such as Newtonian mechanics. This attempt appears quite artificial, I wonder if it could ever work for more complex physical theories such as quantum mechanics, and (as highlighted by the author), despite the book's “Science without numbers” title, science as imagined by Field would NOT be a science without mathematics! Mathematics is built into the theory of space-time.
- other ontological anti-realists, while unable to answer satisfactorily the question of the “unreasonable effectiveness” of mathematics in the scientific endeavor, nevertheless highlight a potential epistemological issue associated with a Platonic position: how can we know something with which we have no causal connection ? How can mathematical knowledge be squared with the abstract nature of mathematical objects ?

The final chapter is dedicated to the philosophical approach with which I feel most comfortable: structuralism. According to this view, we should not be talking about the ontological status of individual mathematical objects, but we should be talking about patterns or STRUCTURES.
Mathematics would be the deductive study of structures, defined as abstract sets of inter-relationships among the individual structure components: the subject of arithmetic would then be the natural number structure, the subject of Euclidean geometry would be the Euclidean space structure. It would be nonsense to contemplate numbers independently of the structure of which they are part.
Such structures, such patterns may be instantiated by many different systems (as is clearly the case in nature) and as such they exist independently of any particular example that implements such structure. The author, who adheres to this current of thought and who is an ontological realist, also provides a convincing explanation of how, from an epistemological perspective, the human mind can, through the process of pattern recognition, progressively apprehend such structure at an increasing level of complexity. The process would start with recognition of simple cardinal structures, and progress all the way to more complicated structures, using also other techniques such as that of “implicit definition” (simultaneous characterization of a number of items in term of their relations to each other).

As you can see, this is a pretty dense, very interesting introductory book that explores several themes and philosophical positions related to the philosophy of mathematics. Of course, given the complexity and vastness of the subject matter, this can only be regarded as the initial stepping stone, but I found this book very informative and helpful, precise and with sufficient level of detail so to be meaningful, and the author achieved this without getting into too much mathematical complexity or philosophical jargon.

Ali says

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Gary says

a nice introduction to the philosophy of mathematics

Alexander says

I've never really enjoyed mathematics and have little aptitude for it, but there are a lot of terrifically interesting philosophical issues involved (e.g., the ontological and epistemic status of numbers, the relationship between mathematics and the natural sciences) that are quite accessible to anyone with a background in philosophy. Shapiro's writing is clear and engaging, and he's generally successful in conveying the philosophical import of topics in mathematics. However, I should note that one will likely get more out of this book if he/she is well-versed in advanced math (especially set theory) and symbolic logic; at times I found myself unable to follow Shapiro because I couldn't understand the math. All in all, however, this was a very worthwhile read.

Jason Gordon says

So far I quite like this book. In its earlier chapters it is quite lucid, but as it becomes more math heavy it becomes a little bit unclear, but the clarity issue is mostly contingent on experience in mathematical logic and philosophical logic. For instance, when defining a bijective function Shapiro writes: "Let us say that two concepts are equinumerous if there is is a one to one correspondence between the objects falling under one and the objects falling under the other." I can see why he may use this definition as the book is geared toward those philosophers who forgot much of their mathematics, but as you can see the definition is a tad clumsy and in my opinion the mathematical definition might have been easier to handle: A set is bijective if it is both surjective or onto (its image equal its codomain or to put it mathematically "for every y in the codomain Y there is at least one x in the domain X such that f(x) = y") and injective (a one to one mapping that preserves distinctness i.e it never maps a distinct element from its domain to the same element of its codomain. In other words every element of the codomain is mapped by at most one element of the domain). The chapter on formalism, particularly the transition from game formalism to deductivism and Hilbert's program, is a chapter that I found grossly unclear (mostly due to the fact that it is difficult to explain metamathematics in laymen's terms) -- Maddy provides a much better explanation of the program and what it is trying to accomplish in her book Realism in Mathematics. However, I will give this chapter a second read and revise this part of the review if necessary. Overall, this is a very good and accessible introduction to [and recapitulation of:] the philosophical positions with in mathematics. Laymen or philosophers lacking experience with mathematical logic will be lost or overwhelmed trying to follow along with the proofs, but they should be able to see the goals each philosophical positions had and the problems they were trying to solve [although to understand why these projects and positions failed one will need to be able to follow the proofs, even if only a little bit, in order to complete the picture:].

Although I'm not finished this a very good book to have in one's library.

Robb Seaton says

Too much philosophy for me.
